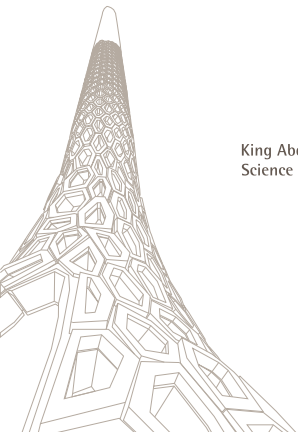


Discussion of Distance-Based Mixture Models for Prior Specification in Spatial Bayesian Analysis



King Abdullah University of
Science and Technology



جامعة الملك عبد الله
للعلوم والتقنية

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Clustering of spatial (areal) data - on the response, a functional or spatial random effect

- Clustering/Grouping (Scan statistics or model-based clustering)
- Boundary detection or wobbling (like Lu and Carlin (2005) and Banerjee and Gelfand (2006))

Contiguity... very problem-specific



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- contiguous
- flexible convex shapes
- very conservative

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Find the divisive boundaries in relative risk over space

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- Hegarty and Barry (2008) - PPM using cohesion functions
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2. In which cases is contiguity useful or necessary?
3. How does the method compare with others?



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Thank you • شكرا



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